UK SENIOR MATHEMATICAL CHALLENGE

Thursday 5 November 2015

Organised by the **United Kingdom Mathematics Trust**

and supported by

Institute and Faculty of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
3. Time allowed: **90 minutes**.
   No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
   Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules**: all candidates start out with 25 marks;
   0 marks are awarded for each question left unanswered;
   4 marks are awarded for each correct answer;
   1 mark is deducted for each incorrect answer.
8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

The United Kingdom Mathematics Trust is a Registered Charity.

http://www.ukmt.org.uk

© UKMT 2015

1. What is $2015^2 - 2016 \times 2014$?

A -2015 B -1 C 0 D 1 E 2015

2. What is the sum of all the solutions of the equation $6x = \dfrac{150}{x}$?

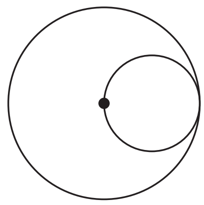
A 0 B 5 C 6 D 25 E 156

3. When Louise had her first car, 50 litres of petrol cost £40. When she filled up the other day, she noticed that 40 litres of petrol cost £50.

By approximately what percentage has the cost of petrol increased over this time?

A 50% B 56% C 67% D 75% E 80%

4. In the diagram, the smaller circle touches the larger circle and also passes through its centre. What fraction of the area of the larger circle is outside the smaller circle?

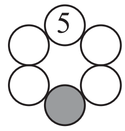


A $\dfrac{2}{3}$ B $\dfrac{3}{4}$ C $\dfrac{4}{5}$ D $\dfrac{5}{6}$ E $\dfrac{6}{7}$

5. The integer n is the mean of the three numbers 17, 23 and $2n$. What is the sum of the digits of n?

A 4 B 5 C 6 D 7 E 8

6. The numbers 5, 6, 7, 8, 9, 10 are to be placed, one in each of the circles in the diagram, so that the sum of the numbers in each pair of touching circles is a prime number. The number 5 is placed in the top circle.



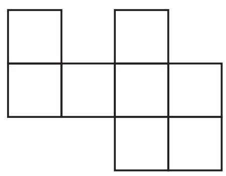
Which number is placed in the shaded circle?

A 6 B 7 C 8 D 9 E 10

7. Which of the following has the largest value?

A $\dfrac{\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)}$ B $\dfrac{1}{\left(\dfrac{\left(\frac{2}{3}\right)}{4}\right)}$ C $\dfrac{\left(\dfrac{\left(\frac{1}{2}\right)}{3}\right)}{4}$ D $\dfrac{1}{\left(\dfrac{2}{\left(\frac{3}{4}\right)}\right)}$ E $\dfrac{\left(\dfrac{1}{\left(\frac{2}{3}\right)}\right)}{4}$

8. The diagram shows eight small squares. Six of these squares are to be shaded so that the shaded squares form the net of a cube.



In how many different ways can this be done?

A 10 B 8 C 7 D 6 E 4

9. Four different straight lines are drawn on a flat piece of paper. The number of points where two or more lines intersect is counted.

Which of the following could **not** be the number of such points?

A 1 B 2 C 3 D 4 E 5

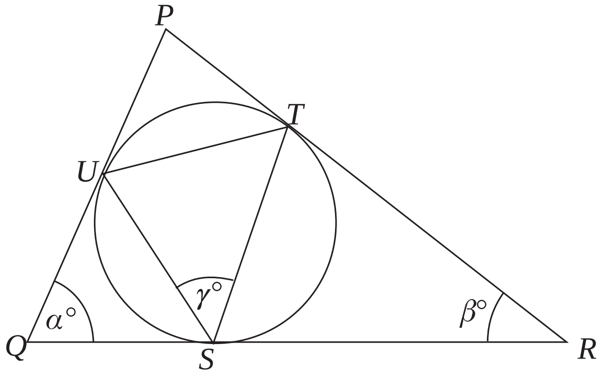
10. The positive integer n is between 1 and 20. Milly adds up all the integers from 1 to n inclusive. Billy adds up all the integers from $n + 1$ to 20 inclusive. Their totals are the same. What is the value of n?

A 11 B 12 C 13 D 14 E 15

11. Rahid has a large number of cubic building blocks. Each block has sides of length 4 cm, 6 cm or 10 cm. Rahid makes little towers built from three blocks stacked on top of each other. How many different heights of tower can he make?

A 6 B 8 C 9 D 12 E 27

12. A circle touches the sides of triangle PQR at the points S, T and U as shown. Also $\angle PQR = \alpha°$, $\angle PRQ = \beta°$ and $\angle TSU = \gamma°$. Which of the following gives γ in terms of α and β?

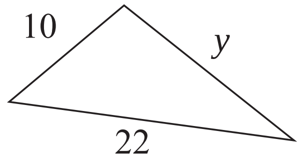


A $\frac{1}{2}(\alpha + \beta)$ B $180 - \frac{1}{2}(\alpha + \beta)$
C $180 - (\alpha + \beta)$ D $\alpha + \beta$
E $\frac{1}{3}(\alpha + \beta)$

13. The Knave of Hearts tells only the truth on Mondays, Tuesdays, Wednesdays and Thursdays. He tells only lies on all the other days. The Knave of Diamonds tells only the truth on Fridays, Saturdays, Sundays and Mondays. He tells only lies on all the other days. On one day last week, they both said, "Yesterday I told lies."
On which day of the week was that?

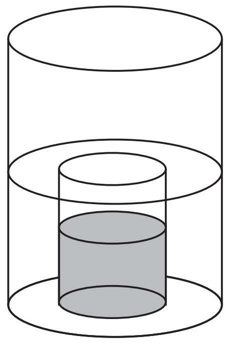
A Sunday B Monday C Tuesday D Thursday E Friday

14. The triangle shown has an area of 88 square units. What is the value of y?



A 17.6 B $2\sqrt{46}$ C $6\sqrt{10}$ D $13\sqrt{2}$ E $8\sqrt{5}$

15. Two vases are cylindrical in shape. The larger vase has diameter 20 cm. The smaller vase has diameter 10 cm and height 16 cm. The larger vase is partially filled with water. Then the empty smaller vase, with the open end at the top, is slowly pushed down into the water, which flows over its rim. When the smaller vase is pushed right down, it is half full of water.
What was the original depth of the water in the larger vase?

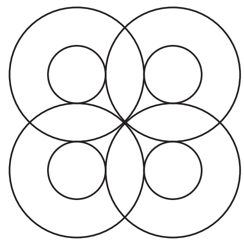


A 10 cm B 12 cm C 14 cm D 16 cm E 18 cm

16. Fnargs are either red or blue and have 2, 3 or 4 heads. A group of six Fnargs consisting of one of each possible form is made to line up such that no immediate neighbours are the same colour nor have the same number of heads. How many ways are there of lining them up from left to right?

A 12 B 24 C 60 D 120 E 720

17. The diagram shows eight circles of two different sizes. The circles are arranged in concentric pairs so that the centres form a square. Each larger circle touches one other larger circle and two smaller circles. The larger circles have radius 1. What is the radius of each smaller circle?

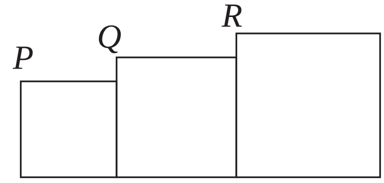


A $\frac{1}{3}$ B $\frac{2}{5}$ C $\sqrt{2} - 1$ D $\frac{1}{2}$ E $\frac{1}{2}\sqrt{2}$

18. What is the largest integer k whose square k^2 is a factor of 10!?
$[10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1.]$

A 6 B 256 C 360 D 720 E 5040

19. Three squares are arranged as shown so that their bases lie on a straight line. Also, the corners P, Q and R lie on a straight line. The middle square has sides that are 8 cm longer than the sides of the smallest square. The largest square has sides of length 50 cm.



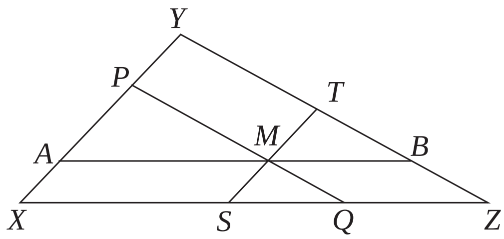
There are two possible values for the length (in cm) of the sides of the smallest square. Which of the following are they?

A 2, 32 B 4, 42 C 4, 34 D 32, 40 E 34, 42

20. A square ink pad has sides of length 1 cm. It is covered in black ink and carefully placed in the middle of a piece of white paper. The square pad is then rotated 180° about one of its corners so that all of the pad remains in contact with the paper throughout the turn. The pad is then removed from the paper. What area of paper, in cm^2, is coloured black?

A $\pi + 2$ B $2\pi - 1$ C 4 D $2\pi - 2$ E $\pi + 1$

21. The diagram shows a triangle XYZ. The sides XY, YZ and XZ have lengths 2, 3 and 4 respectively. The lines AMB, PMQ and SMT are drawn parallel to the sides of triangle XYZ so that AP, QS and BT are of equal length. What is the length of AP?



A $\frac{10}{11}$ B $\frac{11}{12}$ C $\frac{12}{13}$ D $\frac{13}{14}$ E $\frac{14}{15}$

22. Let $f(x) = x + \sqrt{x^2 + 1} + \dfrac{1}{x - \sqrt{x^2 + 1}}$. What is the value of $f(2015)$?

A -1 B 0 C 1 D $\sqrt{2016}$ E 2015

23. Given four different non-zero digits, it is possible to form 24 different four-digit numbers containing each of these four digits. What is the largest prime factor of the sum of the 24 numbers?

A 23 B 93 C 97 D 101 E 113

24. Peter has 25 cards, each printed with a different integer from 1 to 25. He wishes to place N cards in a single row so that the numbers on every adjacent pair of cards have a prime factor in common.
What is the largest value of N for which this is possible?

A 16 B 18 C 20 D 22 E 24

25. A function, defined on the set of positive integers, is such that $f(xy) = f(x) + f(y)$ for all x and y. It is known that $f(10) = 14$ and $f(40) = 20$. What is the value of $f(500)$?

A 29 B 30 C 39 D 48 E 50